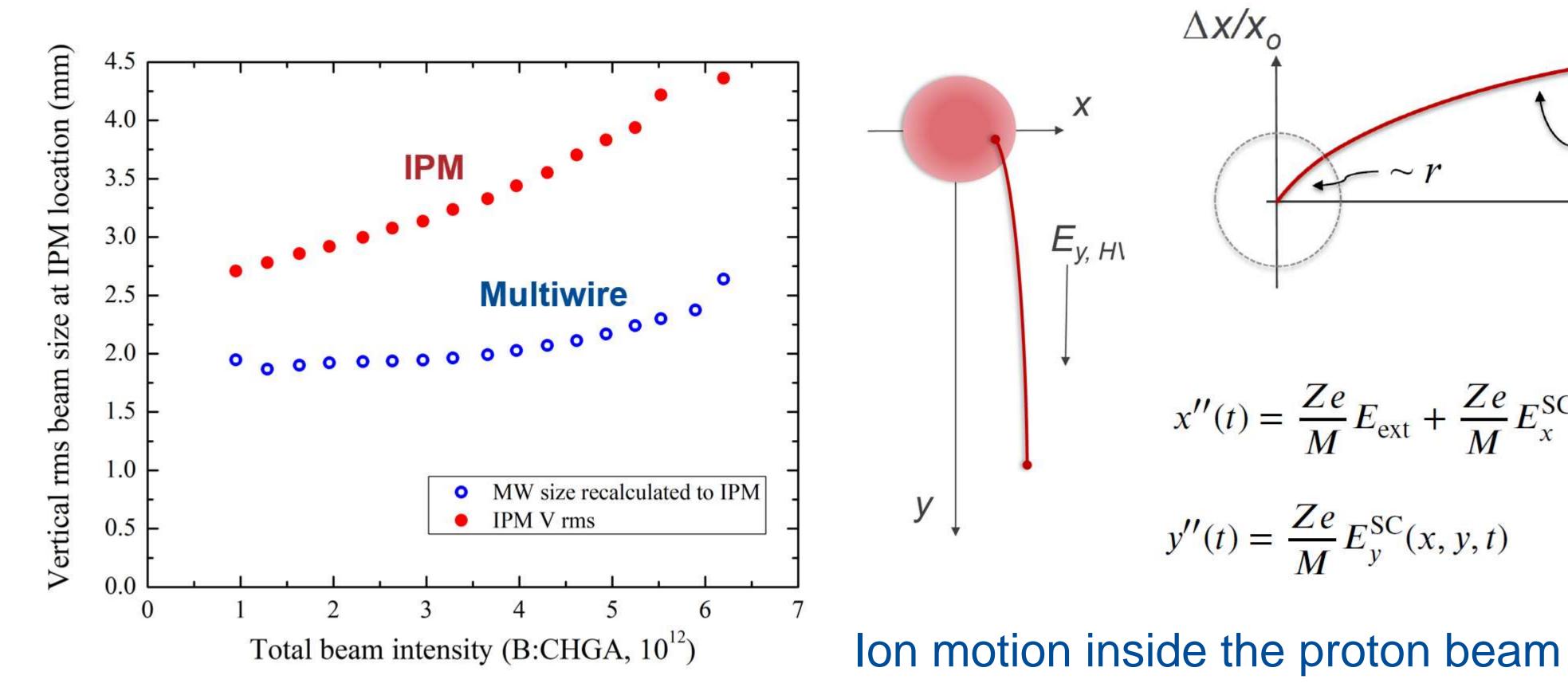
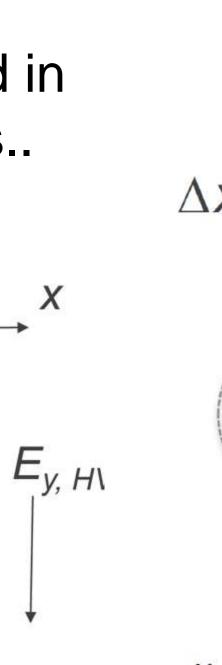
Vladimir Shiltsev, FNAL, PO Box 500, Batavia, IL 60510 USA

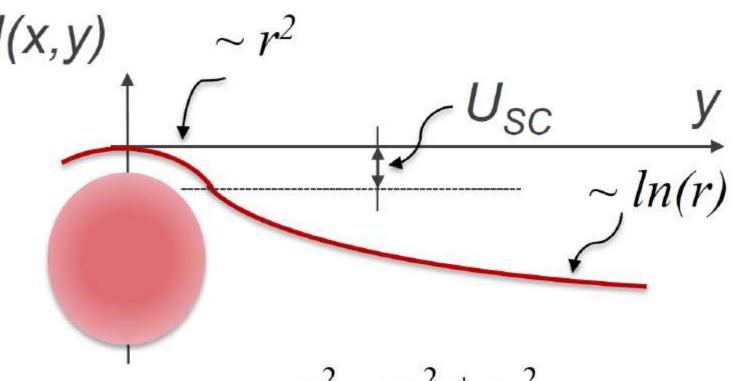
shiltsev@fnal.gov

Abstract:

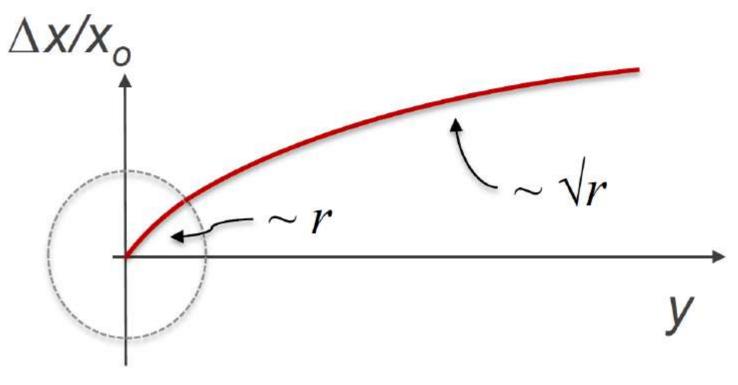
Ionization profile monitors (IPMs) are widely used in accelerators for non-destructive and fast diagnostics of high energy particle beams. At high beam intensities, the spacecharge forces make the measured IPM profiles significantly different from those of the beams. We analyze dynamics of the secondaries in IPMs and develop an effective algorithm to reconstruct the beam sizes from the measured IPM profiles. Efficiency of the developed theory is illustrated in application to the Fermilab 8 GeV proton Booster IPMs..





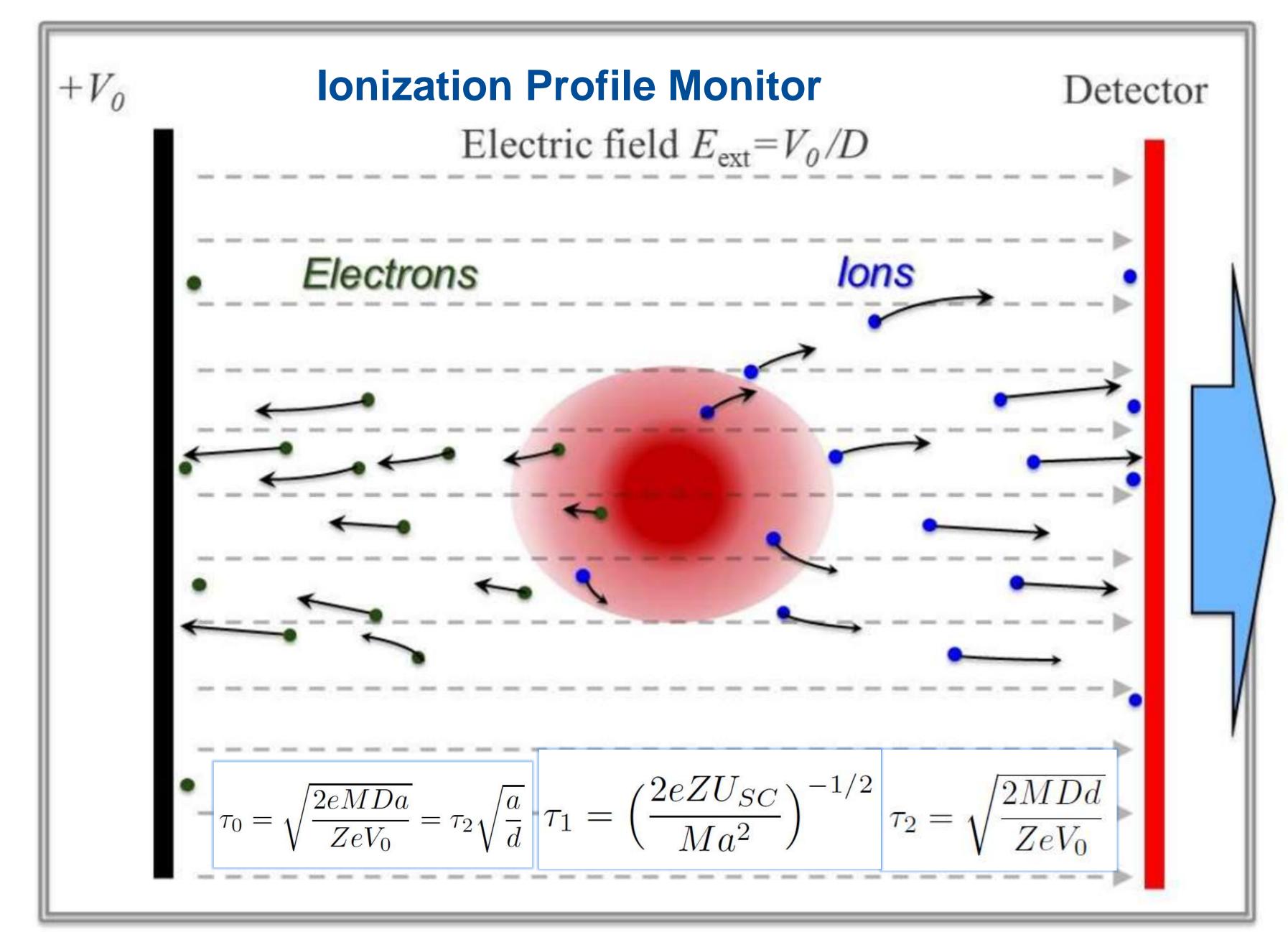


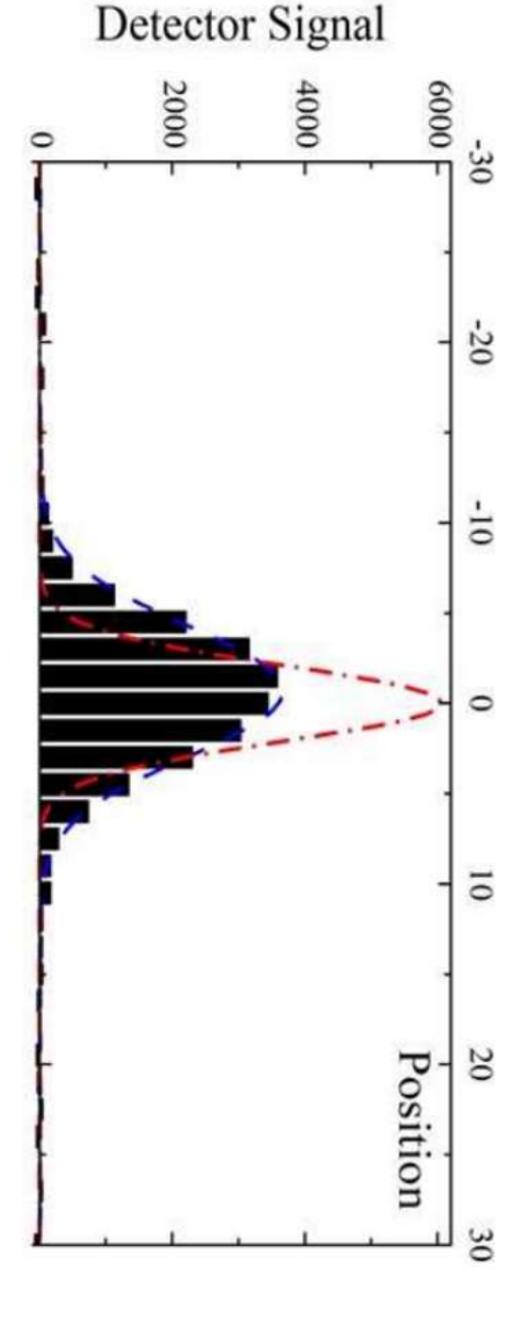
 $r^2 = x^2 + y^2$ $E_v = V_0/D$ $y \approx (ZeE_v/2M) \cdot t^2$



$$x''(t) = \frac{Ze}{M} E_{\text{ext}} + \frac{Ze}{M} E_{x}^{\text{SC}}(x, y, t)$$

$$y''(t) = \frac{Ze}{M} E_y^{SC}(x, y, t)$$





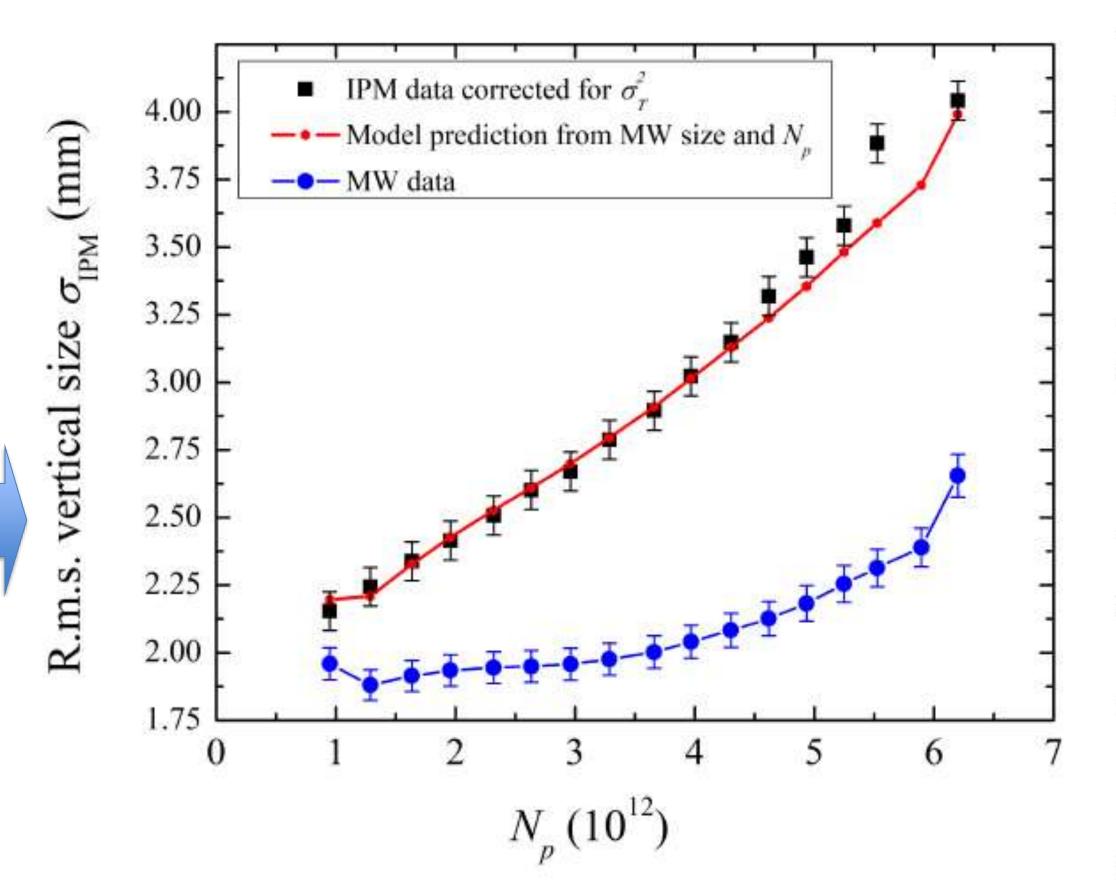
Outside the beam

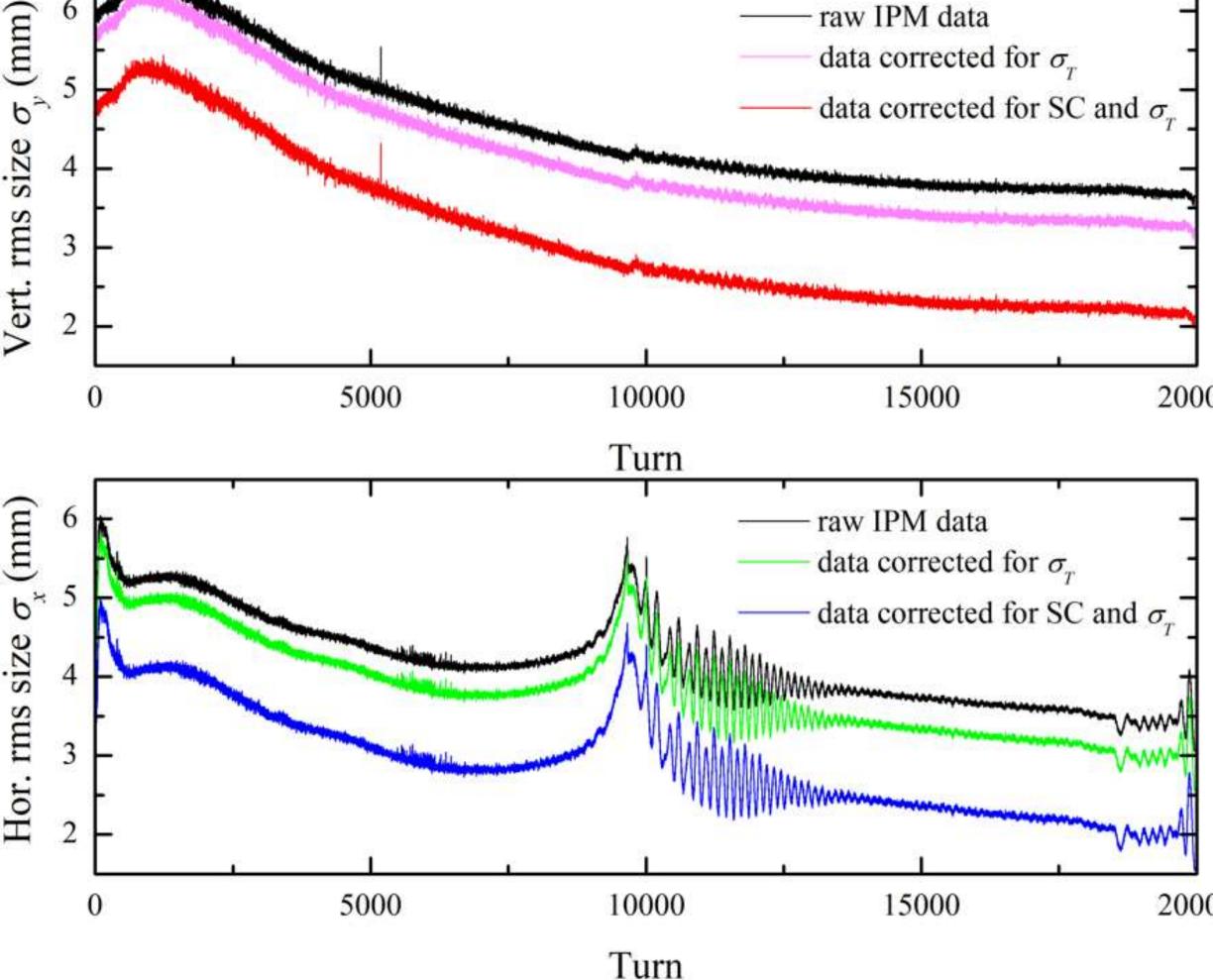
$$y(t) \approx y_0 \text{ch}(t/\tau_1) + v_{0,y} \tau_1 \text{sh}(t/\tau_1)$$

$$y_{[1]}(t) = y_0 \cdot \left[1 + \frac{\tau_0^2}{\tau_1^2} \left(\frac{t}{3\tau_0} \left(\Gamma(\frac{1}{4}) - \Gamma(\frac{1}{4}, \frac{t^4}{\tau_0^4}) \right) - \frac{1}{2} \sqrt{\pi} \operatorname{erf}(\frac{t^2}{\tau_0^2}) + \frac{\tau_0^2}{6t^2} (1 - \exp(-\frac{t^4}{\tau_0^4})) \right) \right]$$

IPM rms size vs original size::

$$\sigma_{m} = \sigma_{0} \cdot h \approx \sigma_{0} \cdot \left[1 + \frac{2U_{SC}}{E_{\text{ext}}\sigma_{0}} \left(\frac{\Gamma(\frac{1}{4})}{3} \sqrt{\frac{d}{\sigma_{0}}} - \frac{\sqrt{\pi}}{2}\right)\right]$$





Reverse (solve) the equation:

$$\sigma_0 pprox rac{\sigma^*}{(1+cN_p/\sigma^{*3/2})(1+lpha c^2N_p^2/\sigma^{*2})} \left[egin{array}{ll} U_{SC}
ightarrow U_{SC}(1+0.8\,t_b/ au_0) \ ext{Thermal velocities} \ \sigma_m^2 = \sigma_0^2 h^2(U_{SC},\sigma_0,V_0,D,d) + \left(rac{4\mathcal{E}_i dD}{ZeV_0}
ight) \end{array}
ight]$$

Also, bunch spacing effect:

$$U_{SC} \to U_{SC}(1 + 0.8 t_b/\tau_0)$$

$$\sigma_m^2 = \sigma_0^2 h^2(U_{SC}, \sigma_0, V_0, D, d) + \left(\frac{4\mathcal{E}_i dD}{ZeV_0}\right)$$

See also Jeff Eldred's talk WEXB08

V.Shiltsev NIM A 986 (2021): 164744

"Beam Size Reconstruction from Ionization Profile Monitors"



